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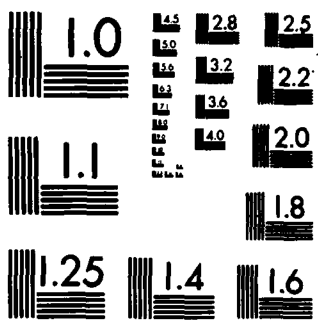

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MULTIVARIABLE L^∞ SENSITIVITY OPTIMIZATION AND HANKEL APPROXIMATION

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Abstract

The problem of designing a feedback compensator to minimize a weighted L^∞ norm of the sensitivity function of a MIMO linear time invariant system is considered. The problem is solved by establishing its equivalence to the different but related problem of multivariable zeroeth order optimal Hankel approximation solved recently by Kung and Lin.

1. Introduction

In this paper the problem of designing a feedback compensator to minimize the sensitivity function of a MIMO linear time invariant system is considered. Sensitivity is measured by a weighted L^∞ norm. The use of a weighted L^∞ norm to measure sensitivity was first proposed by Zames [1]. Zames has argued that a weighted L^∞ norm arises naturally as the optimization criterion in sensitivity minimization problems involving disturbances with variable but bounded power spectra. In contrast, the quadratic norm used in the Wiener-Hopf approach is a meaningful criterion only when the disturbances have a fixed power spectrum [2]. For SISO systems the problem of sensitivity minimization in the L^∞ setting has been solved by Zames and Francis [3-4]. Safonov and Chen [5] provide a solution for the MIMO case when the sensitivity is constrained to be decoupled (i.e., diagonal). Francis, Helton and Zames [6] and Chang and Pearson [7] have solved the MIMO problem without decoupling constraints. Here we present a different solution for the MIMO case which establishes important links between L^∞ sensitivity optimization and Hankel norm optimal approximation theory.

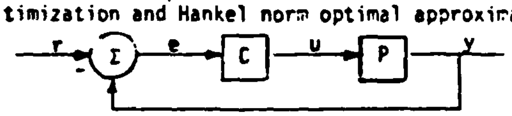

Fig. 1

For the feedback configuration of Fig. 1, the sensitivity function is $S = (I+PC)^{-1}$ which is the transfer function from the reference signal r to the error signal e. The weighted sensitivity function we consider is $T + W_\ell S W_r^{-1}$ where W_ℓ and W_r are appropriately chosen weighting matrices. The optimization problem is solved in essentially three stages: 1) The class of sensitivity functions for which the closed loop system is stable is parameterized by the fractional representation approach [8]. 2) The problem is translated into one of optimal H^∞ interpolation. 3) A solution to the interpolation problem is given by establishing its equivalence to the different but related problem of zeroeth order optimal Hankel norm model approximation solved by Kung and Lin [9,10], who extended the earlier SISO results of Adamjn, Arov and Krein [11].

Notation is introduced in section 2. In section 3 realizable sensitivity functions are defined and parameterized. In section 4 the sensitivity optimization problem is shown to be equivalent to an optimal interpolation problem. Its links with the optimal Hankel norm approximation problem are established in section 5. Concluding remarks are presented in section 6.

2. Notation and Preliminaries

We let H^∞ denote the space of matrices whose elements are functions analytic and bounded in the r.h.p. and let L^∞ denote the space of matrices whose elements are functions bounded on the $j\omega$ - axis. The space H^∞ is a subspace of L^∞. The norm of $G(s) \in L^\infty$ is defined as

$$\|G\|_\infty = \sup_\omega \bar{\sigma}(G(j\omega)) \tag{2.1}$$

where $\bar{\sigma}(A)$ denotes the largest singular value of A. A matrix function is *stable* if its elements are analytic in $\mathrm{Re}(s) \geq 0$. The elements of H^∞ are stable and proper.

A matrix A(s) is *all-pass* if

$$A^*(j\omega)\, A(j\omega) = I \quad \text{for all } \omega \tag{2.2}$$

where $A^*(s)$ denotes $A^T(-s)$. A(s) is *inner* if it is all pass and stable. B(s) is *min. phase* if it has no r.h.p. zeros and *outer* if it is min-phase, stable and proper.

Two elements of a ring are right (left) coprime if their only common right (left) factors in the ring have inverses in the ring. We will apply the concept of coprimeness to the rings of rational, stable and rational, stable, proper matrices.

The plant and the compensator transfer functions are denoted by P and C respectively. P is assumed to be an nxn real, rational, proper matrix having $m \geq 0$ distinct r.h.p. zeros $z_1, z_2, \ldots, z_m$, and no poles or zeros on the $j\omega$ - axis. The sensitivity function is $S = (I+PC)^{-1}$ and the weighted sensitivity function is $T = W_\ell S W_r^{-1}$. The matrices W_ℓ and W_r reflect relative frequency weightings on the sensitivity function and the reference signals in the sense that

$$\|W_\ell S W_r^{-1}\|_\infty = \sup_{0 \neq x \in L_2} \frac{\|W_\ell S x\|_2}{\|W_r x\|_2}$$

where $\|\cdot\|_2$ denotes the L^2 norm. We assume W_ℓ and W_r^{-1} to be rational, stable, proper and min. phase and of unit norm. The stable, min. phase condition is not a restriction since if, say, $W_\ell(s)$ is non-min. phase then we can take in its place the min. phase spectral factor of $W_\ell^T(-s)\, W_\ell(s)$ which gives rise to the same frequency weighting as $W_\ell(s)$ [3, 12].

3. Realizable Sensitivity Functions

In this section we characterize those sensitivity functions which correspond to a stable closed-loop (c.l.) system.

Def: A rational[1] sensitivity function S is realizable iff $S = (I+PC)^{-1}$ for some C which stabilizes the closed-loop system.

Lemma 1: There exist rational, stable, matrices B_r, D_r, N_ℓ, D_ℓ, U_ℓ and V_ℓ, with B_r, D_ℓ inner and U_ℓ, V_ℓ proper such that

$$B_r W_\ell P = \frac{a^*}{a} D_r^{-1}$$

$$W_r P = D_\ell^{-1} N_\ell$$

$$N_\ell U_\ell + D_\ell V_\ell = I$$

where

$$a^*(s) \triangleq \prod_{i=1}^{m} (s - z_i).$$

Proof: Since P is a rational, proper matrix, it admits of right and left fractional representations

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[1] All matrices used in this section are rational.

$$P = N_{r1} D_{r1}^{-1} = D_{\ell 1}^{-1} N_{\ell 1} \tag{3.1}$$

where N_{r1}, D_{r1} are right coprime and $N_{\ell 1}$, $D_{\ell 1}$ are left coprime in the ring of rational, stable, proper matrices [13]. $W_\ell N_{r1}$ is rational, stable, proper and can be factored as $W_\ell N_{r1} = A_i A_o$, where A_i is inner and A_o is *outer* [12]. Define $a^*(s) \triangleq \prod_{i=1}^{m} (s - z_i)$ and $B_r \triangleq \frac{a^*(s)}{a(s)} A_i^{-1}$. The r.h.p. zeros of A_i, being the r.h.p. zeros of N_{r1} and in turn the r.h.p. zeros of P, are contained in $a^*(s)$. Hence B_r is stable. Further, $\frac{a^*}{a}$ and A_i being all-pass B_r is all-pass. Hence, B_r is inner. We can now write $W_\ell N_{r1} = \frac{a^*}{a} B_r^{-1} A_o$. A_o being outer we can factor $D_{r1} = D_r A_o$, D_r stable. Then $B_r W_\ell P = \frac{a^*}{a} D_r^{-1}$.

Similary factor $D_{\ell 1} W_r^{-1} = A_{o1} \cdot D_\ell$, where A_{o1} is outer and D_ℓ is inner. Again, A_{o1} being outer we can factor $N_{\ell 1} = A_{o1} \cdot N_\ell$, N_ℓ stable. Then $W_r P = D_\ell^{-1} N_\ell$.

Since $N_{\ell 1}$, $D_{\ell 1}$ are left coprime in the ring of rational, stable, proper matrices, there exist rational, stable, proper $U_{\ell 1}$, $V_{\ell 1}$ which satisfy the Bezout identity

$$N_{\ell 1} U_{\ell 1} + D_{\ell 1} V_{\ell 1} = I \tag{3.2}$$

[13]. If we substitute $N_{\ell 1} = A_{o1} N_\ell$ and $D_{\ell 1} = A_{o1} D_\ell W_r$ then $A_{o1} N_\ell U_{\ell 1} + A_{o1} D_\ell W_r V_{\ell 1} = I$ which is equivalent to $N_\ell U_{\ell 1} A_{o1} + D_\ell W_r V_{\ell 1} A_{o1} = I$. Now $U_\ell \triangleq U_{\ell 1} \cdot A_{o1}$, stable, proper and $V_\ell \triangleq W_r V_{\ell 1} A_{o1}$, stable, satisfy

$$N_\ell U_\ell + D_\ell V_\ell = I \tag{3.3}$$

Eq. (3.3) implies that $D_\ell V_\ell$ is proper. Since D_ℓ is inner this implies that V_ℓ is proper. Q.E.D.

Lemma 2: S is realizable iff

$$T \triangleq W_\ell S W_r^{-1} = (W_\ell W_r^{-1} V_\ell - \frac{a^*}{a} B_r^{-1} X) D_\ell \tag{3.4}$$

for some rational, stable X.

Proof: Note that (3.1) is a coprime fractional representation of P and (3.2) is the corresponding left Bezout identity in the ring of rational, stable matrices also. Hence from [8], S is realizable iff

$$S = (N_{r1} Z + V_{\ell 1}) D_{\ell 1} \tag{3.5}$$

for some rational, stable Z. If we substitute for N_{r1}, $V_{\ell 1}$ and $D_{\ell 1}$ from the proof of Lemma 1, (3.5) is equivalent to

$$T = W_\ell S W_r^{-1} = (W_\ell W_r^{-1} V_\ell - \frac{a^*}{a} B_r^{-1} X) D_\ell$$

for $X = - B_o Z A_{o1}$. B_o and A_{o1} being rational, outer, X is rational, stable iff Z is rational, stable. Hence, S is given by (3.5) for some rational, stable Z iff T is given by (3.4) for some rational, stable X. Q.E.D.

Eq. (3.4) gives a parametrization of rational, stable *realizable* weighted sensitivity functions T in terms of rational, stable X. In the following section we will also consider T's which are given by (3.4) but which are rational, stable and proper or only stable and proper. The former are parameterized by rational, stable, proper X and the latter by stable, proper X, i.e., by $X \in H^\infty$. Q.E.D.

4. The Interpolation Problem

In this section we show that the characterization (3.4) of *realizable sensitivity functions* translates the sensitivity minimization problem into one of optimal H^∞-interpolation. Our objective is to solve

Problem 1: Find a rational compensator C(s) to minimize

$$\|T\|_\infty = \|W_\ell (I + PC)^{-1} W_r^{-1}\|_\infty \tag{4.1}$$

subject to the constraint that the closed-loop system of Fig. 1 is stable. □

As a consequence of Lemma 2, problem 1 involves *minimization over the set of weighted sensitivity functions parameterized as*

$$T = (W_\ell W_r^{-1} V_\ell - \frac{a^*}{a} B_r^{-1} X) D_\ell \tag{4.2}$$

X rational, stable. If we take X = 0 then $T = W_\ell W_r^{-1} V_\ell D_\ell$ is proper and has finite norm. Consequently, the minimun attained in problem1 is finite. Also, $\|T\|_\infty = \infty$ if T is improper. *Hence we can restrict ourselves to minimizing over proper T's only.* We can also relax the constraint that T be rational since the optimum T turns out to be rational [4,5]. With these modifications and recalling that stable, proper T's are parameterized by $X \in H^\infty$ we have an equivalent problem.

Problem 2:

$$\min_{X \in H^\infty} \|T(X)\|_\infty \tag{4.3}$$

where

$$T(X) = (W_\ell W_r^{-1} V_\ell - \frac{a^*}{a} B_r^{-1} X) D_\ell \qquad \square$$

If we define

$$K \triangleq B_r W_\ell W_r^{-1} V_\ell \tag{4.4}$$

$$Y \triangleq K - \frac{a^*}{a} X \tag{4.5}$$

then

$$Y = B_r T D_\ell^{-1}. \tag{4.6}$$

Since B_r, D_ℓ are inner

$$\|Y\|_\infty = \|T\|_\infty \tag{4.7}$$

and problem 2 is equivalent to

Problem 3:

$$\min_{X \in H^\infty} \|K - \frac{a^*}{a} X\|_\infty \tag{4.8}$$
□

If Y^o achieves the minimum in (4.8), then the optimal sensitivity is

$$S^o = (B_r W_\ell)^{-1} Y^o (D_\ell W_r) \tag{4.9}$$

and the optimizing compensator is

$$C^o = P^{-1}(S^{o-1} - I) \tag{4.10}$$

Remark: Problem 3 is solved by Sarason [15] in the context of H^∞-interpolation. Francis and Zames [4] show the sensitivity minimization problem to be equivalent to problem 3 for the SISO case and Chang and Pearson [7] for the MIMO case. In both cases Sarason's results are finally used to obtain the minimal sensitivity. Here we take a different approach. We convert problem 3 into a Hankel-norm approximation problem and thus establish an important link between L^∞ sensitivity minimization and optimal Hankel approximation.

The following Lemma shows that the requirement that Y be of the form $Y = K - \frac{a^*}{a} X$ for some $X \in H^\infty$ is in fact an interpolation constraint on Y.

Lemma 3: (a) $Y = K - \frac{a^*}{a} X$ for some $X \in H^\infty$ iff $Y(z_i) = K(z_i)$, $1 \le i \le m$.

(b)
$$\begin{aligned} Y^o &= \mathrm{Min}\{\|Y\|_\infty \mid Y = K - \tfrac{a^*}{a}X,\ X \in H^\infty\} \\ &= \mathrm{Min}\{\|Y\|_\infty \mid Y = A - \tfrac{a^*}{a}X,\ X \in H^\infty\} \end{aligned} \tag{4.11}$$

for only A satisfying $A(z_i) = K(z_i)$, $1 \le i \le m$.

Proof: (a) If $Y = K - \frac{a^*}{a}X$ for some $X \in H^\infty$ then for $1 \le i \le m$, $Y(z_i) = K(z_i)$ since $a^*(z_i) = 0$. Conversely, if $Y(z_i) = K(z_i)$ for $1 \le i \le m$ then $\frac{a}{a^*}(Y-K)$ is stable and proper, i.e., $\frac{a}{a^*}(Y-K) \in H^\infty$. Hence $Y = K - \frac{a^*}{a}X$ for

some $X \in H^\infty$.

(b) From part (a), $Y = K - \frac{a^*}{a}X$ for some $X \in H^\infty$ iff $Y(z_i) = K(z_i)$, $1 \le i \le m$. Similarly $Y = A - \frac{a^*}{a}X_1$ for some $X_1 \in H^\infty$ iff $Y(z_i) = A(z_i)$, $1 \le i \le m$. But since $A(z_i) = K(z_i)$, $1 \le i \le m$, $Y = K - \frac{a^*}{a}X$ for some $X \in H^\infty$ iff $Y = A - \frac{a^*}{a}X_1$ for some $X_1 \in H^\infty$. Eq. (4.11) then follows.

As a consequence of Lemma 3, problem 3 is essentially an optimal H^∞-interpolation problem, i.e., one of finding a $Y \in H^\infty$ of minimum norm which satisfies the r.h.p. interpolation constraints $Y(z_i) = K(z_i)$, $1 \le i \le m$.

We next present the interpolation problem in a form which leads to a link with a Hankel approximation problem.

Theorem 1: Problem 3 has the same solution Y^0 as

Problem 4:

$$\text{Min}\{\|Y\|_\infty \mid Y = \frac{a^*}{a}(H^* - X),\ X \in H^\infty\} \tag{4.12}$$

where

$$H^*(s) \triangleq \sum_{i=1}^{m} [\frac{a}{a^*}(s-z_i)K]_{s=z_i} \frac{1}{(s-z_i)} \tag{4.13}$$

Proof:

$$\frac{a^*}{a} H^*|_{s=z_i} = K(z_i),\ 1 \le i \le m.$$

Then from Lemma 3 (b) Y^0 is a solution of (4.12) iff it is a solution of (4.8). Hence problems 3 and 4 have the same solution Y^0. Q.E.D.

5. Zeroeth-Order Hankel Norm Approximation

The solution to the optimization problem 4 has been found by Kung and Lin [9, 10] as an intermediate step in the solution of the optimal Hankel norm approximation problem. With every transfer function $G(s) \in L^\infty$ *there is associated a Hankel operator* $\Gamma(G)$ defined as

$$(\Gamma(G)u)(t) = \int_0^\infty G_c(t+\tau)\, u(\tau) d\tau \quad 0 < t < \infty \tag{5.1}$$

where $G_c(t) \triangleq \mathcal{L}^{-1}[G(s)]_-$, $[\cdot]_-$ being the projection operator which retains only the stable part of the partial fraction expansion of its argument. The rank of $\Gamma(G)$ is the number of its nonzero singular values. It equals the order of $[G(s)]_-$ which is defined as the dimension of its minimal state space realization. It equals the number of l.h.p. poles of $G(s)$. In the Hankel norm model approximation problem the objective is to obtain a lower order approximation $G_a(s)$ to $G(s)$ so that the Hankel norm of the error $E(s) \triangleq G(s) - G_a(s)$ is small. *The Hankel norm of $G(s)$ is defined as the largest singular value of the associated Hankel operator* $\Gamma(G)$.

The following result from Kung and Lin [9, 10] relates the bounds on the error to the order of the approximation.

Lemma 4: If $[G_a(s)]_-$ has order $K \ge 0$, then

$$\sigma_{k+1}(G) \le \bar{\sigma}[\Gamma(G) - \Gamma(G_a)] \le \|G - G_a\|_\infty \tag{5.2}$$

where $\sigma_{k+1}(G)$ denotes the $(k+1)^{st}$ largest singular value of $\Gamma(G)$. Further, there exists a $G_a^0(s)$ of order $[G_a^0(s)]_- = k$ for which equalities are obtained in (5.2) and $E^0 \triangleq G - G_a^0 = \sigma_{k+1}(G)\,\tilde{E}$ where $\tilde{E}$ is all-pass and rational. G_a^0 is called the optimal k^{th} order Hankel approximant of $G(s)$ and E^0 the corresponding minimal error.

We will make use of this Lemma to show that a solution to problem 4 can be obtained via the optimal zeroeth order Hankel approximation of $H(s)$.

Theorem 2: If $H_a^0(s)$ is an optimal zeroeth order Hankel approximant of $H(s)$ and E^0 the corresponding error, then

$$\text{(a)}\ \underset{X \in H^\infty}{\text{Min}} \|\frac{a^*}{a}(H^* - X)\|_\infty = \|H - H_a^0\|_\infty = \bar{\sigma}[\Gamma(H) - \Gamma(H_a^0)] = \sigma_1(H) \tag{5.3}$$

(b) $Y^0 = \frac{a^*}{a}(H^* - H_a^{0*}) = \frac{a^*}{a} E^{0*}$

(c) $Y^0 = \sigma_1(H)\,\tilde{Y}$ where $\tilde{Y}$ is all-pass.

Proof: (a) If we take $k = 0$ in Lemma 4 then H_a^0 is the optimal zeroeth order Hankel approximant of $H(s)$ and

$$\sigma_1(H) = \bar{\sigma}[\Gamma(H) - \Gamma(H_a^0)] = \|H - H_a^0\|_\infty .$$

Since $\frac{a^*}{a}$ is all-pass

$$\sigma_1(H) = \|H - H_a^0\|_\infty = \|H^* - H_a^{0*}\|_\infty = \|\frac{a^*}{a}(H^* - H_a^{0*})\|_\infty .$$

$\|H - H_a^0\|_\infty = \sigma_1(H)$ being finite $H - H_a^0$ is proper and H being proper H_a^0 is proper. The order of H_a^0 being zero, H_a^0 has no l.h.p. poles. Hence H_a^{0*} has no r.h.p. poles and $X^0 \triangleq H_a^{0*}$ is stable and proper. Thus $X^0 \in H^\infty$ yields $\sigma_1(H) = \|\frac{a^*}{a}(H^* - X^0)\|_\infty$. We claim that X^0 achieves the minimum in (5.3). For, if there existed an $X \in H^\infty$ such that $\|\frac{a^*}{a}(H^* - X)\|_\infty < \sigma_1(H)$ then $\|H - X^*\|_\infty < \sigma_1(H)$ and $H_a \triangleq [X^*]_-$ *is a zeroeth order approximant* of $H(s)$ which violates Eq. (5.2) of Lemma 4. Thus, $\sigma_1(H) = \|\frac{a^*}{a}(H^* - X^0)\|_\infty = \underset{X \in H^\infty}{\text{Min}} \|\frac{a^*}{a}(H^* - X)\|_\infty$

(b) $Y^0 = \frac{a^*}{a}(H^* - X^0) = \frac{a^*}{a}(H^* - H_a^{0*}) = \frac{a^*}{a} E^{0*}$.

(c) Since $E^0 = \sigma_1(H)\tilde{E}$ where $\tilde{E}$ is all-pass $Y^0 = \frac{a^*}{a} E^{0*} = \sigma_1(H)\tilde{Y}$ where $\tilde{Y} \triangleq \frac{a^*}{a}\tilde{E}^*$ is all-pass since $\frac{a^*}{a}$ and $\tilde{E}$ are all-pass. Q.E.D.

Hence we have obtained

$$Y^0 = \frac{a^*}{a} E^{0*}, \tag{5.4}$$

the solution to problem 4, in terms of the *minimal* zeroeth order Hankel error E^0 of $H(s)$. It gives us the optimal weighted sensitivity

$$T^0 = B_r^{-1}\, Y^0\, D_\ell . \tag{5.5}$$

Since B_r, D_ℓ are all-pass, T^0 is of the form

$$T^0 = \sigma_1(H)\,\tilde{T} \tag{5.6}$$

where $\tilde{T}$ is all-pass. Note that all our optimal solutions, H_a^0, X^0, E^0, Y^0, T^0 are rational [9, 10] which justifies the assumption we made in defining problem 2.

Remark: The all-pass form of the optimal weighted sensitivity is in agreement with the SISO results obtained in [3,4]. It is likely that the optimal compensator given by (4.10) will turn out to be improper. In that case we can approximate the improper compensator by a proper one by using high frequency attenuation as in [3,4] and the approximation can be made to yield sensitivities arbitrarily close to the optimal over any finite bandwidth.

To compute $\sigma_1(H)$ and/or the optimal zeroeth order Hankel approximant of $H(s)$ one may use the algorithm given by Lin and Kung [9, 10]. The algorithm involves computing the largest singular value of $\Gamma(H)$ and solving a matrix polynomial equation and an algebraic riccati equation. In order to obtain $H(s)$ we have to compute B_r and V_ℓ (Eq. (4.4), (4.13)). This requires left and right coprime *fractional representation* of P, solving a left Bezout identity and inner-outer factorizations of stable rational matrices. An algorithm for obtaining a coprime fractional representation of rational, proper matrices and solving the corresponding Bezout identity is given in [13]. An algorithm for

computing inner-outer factorizations is given in [14].

If one desires to compute only the minimal value $\sigma_1(H)$ of the cost (4.12), another simple approach is afforded by the identity

$$\sigma_1(H) = (\lambda\max(M_o M_c))^{\frac{1}{2}}$$

where $\lambda\max(\cdot)$ denotes the greatest eigenvalue and M_o and M_c are the observability and reachability grammians of any state-space realization of H(s) [16].

6. Conclusions

The main contribution of this paper lies in establishing a link between the multivariable L^∞ sensitivity optimization problem and the optimal Hankel approximation problem. As in [3,4] we have used the fractional representation approach to parameterize the set of realizable sensitivity functions. With this parameterization, the problem of minimizing sensitivity over the set of stabilizing compensators leads to an optimal H^∞-interpolation problem.

We have solved this problem by solving the equivalent optimal zeroeth order Hankel approximation problem. The optimal sensitivity is related to the optimal Hankel error in a simple way, and the optimal Hankel error can be computed by using an algorithm of Lin and Kung [9,10].

The approach taken in [4,5,6] to solve the optimal H^∞-interpolation problem is based on Sarason's theory [15] and makes use of the Nevanlinna algorithm to compute the optimal sensitivity. If we exploit the relationship established between optimal H^∞-interpolation and optimal Hankel approximation we can use the Nevanlinna algorithm for computing optimal zeroeth order Hankel approximants as well. This points to the possibility of using the Nevanlinna algorithm in higher order Hankel approximation problems as well, assuming that these problems could be shown to be equivalent to some H^∞-interpolation problem. This possibility is not a remote one given the fundamental relationship that exists between H^∞-interpolation and Hankel approximation.

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The problem of designing a feedback compensator to minimize a weighted L-infinity norm of the sensitivity function of a MIMO linear time invariant system is considered. The problem is solved by establishing its equivalence to the different but related problem of multivariable zeroeth order optimal Hankel approximation solved recently by Kung and Lin.

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